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Interaction of Optical Phonons and Cyclotron Waves in Semiconducting Plasmas

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The interaction of optical phonons and cyclotron waves is considered for semiconductors in a homogeneous dc magnetic field. The charge-density fluctuation is calculated and the splitting of the coupled phonon-cyclotron harmonic waves is obtained. The reflectivity in the Faraday configuration has been calculated when the transverse-phonon frequency is about equal to the cyclotron frequency. The drastic change in the reflectivity coefficient makes it a useful tool for studying the phonon–cyclotron-wave interaction.

I. INTRODUCTION

Collective excitations in semiconductors, and their interaction, have been studied intensively during the past few years. In particular, the availability of intense monochromatic sources in the infrared has made possible Raman scattering from narrow-gap semiconductors. More specifically, light scattering experiments have been performed in weakly doped GaAs,[1,2] InAs, and InSb.[3,4] In the preceding the spectrum of interacting plasmon phonon has been observed as well as the spectrum of interacting plasma-hybrid and cyclotron modes.[5] In experiments on light scattering at small angles the polariton spectrum has been observed and an attempt to obtain information on plasma polariton for a plasma in a magnetic field has been made.[4]

In previous papers[6] considering small-angle light scattering, we reported the feasibility of detecting the interaction of phonons with cyclotron modes which had not been discussed earlier. We consider here two different experimental situations by which coupled phonon-cyclotron modes can be observed.

In large-angle Raman scattering with momentum transfer $\vec{k}$ perpendicular to $\vec{B}_0$, the static magnetic field, the cross section is proportional to the poles of the inverse longitudinal dielectric function, $\epsilon_{\vec{k}\omega}$. Here the scattered light intensity will reside in the phonon and plasma lines.

The scattered intensity at the cyclotron harmonic modes is smaller than the intensity of the phonon line by a factor of $(k/k_D)^2$ and thus is difficult to detect. However, when the frequency of the second-harmonic mode, $2\omega_c$, is "about" equal to the longitudinal phonon frequency ω_l, the two modes interact strongly with each other and the mixed cyclotron-phonon modes will appear with nearly equal intensity. The splitting frequency is proportional to k and is easier to detect than the k^2-dependent part of the cyclotron mode frequency at small k. The calculations are presented in Sec. II.

Reflectivity considerations for a plasma-phonon system indicate no electromagnetic propagation in two frequency bands: when ω is smaller "roughly" than ω_p[7] and when ω is "roughly" between ω_t and ω_l (ω_t is the transverse phonon frequency).[8] These forbidden gaps are strongly affected in the presence of dc magnetic field for right-hand circularly light due to the interaction of the phonon with the cyclotron mode. This interaction is most pronounced when $\omega_c \sim \omega_t$ (transverse wave interaction). It can be observed from reflectivity measurements which are discussed in Sec. III.

II. CALCULATION OF THE RAMAN CROSS SECTION

To calculate the light scattering cross section from the conduction electrons we consider an interacting single-component electron gas immersed in a polarizable medium consisting of the lattice and the valence electrons. This is a reasonably realistic model for many semiconducting plasma-phonon systems when the wave number of the excitations is much smaller than an inverse lattice spacing.

For almost-transparent plasmas the radiation field interacts weakly with the systems and the cross section may be computed in the Born approximation to yield

$$\frac{d\sigma}{d\omega\, d\Omega} = V\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}} \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt\, e^{i\omega t}\langle n_k(t) n_{-k}\rangle \ . \qquad (1)$$

Here Ω is the solid angle,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Th}} \equiv \frac{e^4}{m^{*2}c^4}(\vec{\epsilon}_1 \cdot \vec{\epsilon}_2)^2$$

is the Thompson cross section and $\langle n_k(t) n_{-k}\rangle$ is the thermodynamic time-dependent density correlation. In arriving at Eq. (1) we have neglected the $\vec{p}\cdot\vec{A}$

terms in the coupling of the electromagnetic field to the conduction electrons which is a good approximation for nonrelativistic plasmas. However, the $\vec{p}\cdot\vec{A}$ terms in the coupling of the electromagnetic field to the valence electrons have been retained, and manifest themselves in renormalizing the conduction mass to be m^*, the electron effective mass, which for simplicity we consider to be a scalar.

For system in thermal equilibrium the Fourier transform of the density correlation function in Eq. (1) is simply related to the response function to an external longitudinal potential. For wave number k, such that $\omega/kc \ll 1$, the phase velocity of the density fluctuations is much smaller than the speed of light and the response function is given by the in-inverse of the longitudinal dielectric function. We thus obtain

$$\frac{d\sigma}{d\omega\, d\Omega} = V\left(\frac{d\sigma}{d\Omega}\right)_{\text{Th}} \frac{1}{\pi}\frac{\hbar}{1-e^{-\beta\hbar\omega}}\frac{\hbar^2}{4\pi e^2}\,\text{Im}\frac{1}{\epsilon_{\vec{k}\omega}}\,, \quad (2)$$

where the effects of the magnetic field and the "phonon" field are buried in the behavior of the dielectric function $\epsilon_{\vec{k}\omega}$. It is worthwhile mentioning here that we explicitly assume "long-lived" phonons (represented by the polarizability of the lattice) and will consider lifetime effects due to electronic collisions, which is a realistic situation for most semiconductors. Thus linewidths obtained using our phenomenological lifetimes in the limit $\omega_p \to 0$ are outside the validity of our theory.

To calculate the spectrum of scattered radiation from a nondegenerate plasma, we let $h \to 0$ and $hT/E_F \to \infty$ in Eq. (2) and obtain

$$\frac{d\sigma}{d\omega\, d\Omega} = N\left(\frac{d\sigma}{d\Omega}\right)_{\text{Th}} \frac{1}{\pi}\left(\frac{k}{k_D}\right)^2 \frac{1}{\omega}\,\text{Im}\frac{1}{\epsilon_{\vec{k}\omega}}\,, \quad (3)$$

where $k_D^2 = 4\pi n e^2 \beta = \omega_p^2/v_{\text{Th}}^2$ is the Debye wave number squared and N is the total number of conduction electrons. The dielectric function $\epsilon_{\vec{k}\omega}$, for the particular geometry of interest, i.e., $\vec{k} \perp \vec{B}_0$, is given by[9]

$$\epsilon_{\vec{k}\omega} = \epsilon_\infty \frac{\omega^2-\omega_l^2}{\omega^2-\omega_t^2} - \frac{\tilde{\omega}}{\omega}\frac{\omega_p^2}{\omega_c^2}\frac{1}{\lambda}\left[\left[e^{-\lambda}I_0(\lambda)-1\right] + 2\left(\frac{\tilde{\omega}}{\omega_c}\right)^2 \sum_{n=1}^{\infty}\frac{e^{-\lambda}I_n(\lambda)}{(\tilde{\omega}/\omega_c)^2-n^2}\right]. \quad (4)$$

Here ω_p and ω_c are the electron plasma and cyclotron frequencies, respectively, $\lambda = k^2 v_{\text{Th}}^2/\omega_c^2$, the I_n's are the Bessel functions of the second kind, and $\omega = \omega + i\tau$, where τ is the phenomenological electronic lifetime.

The poles of the scattered intensity, as can be seen from Eqs. (3) and (4), are given by the zeros of $\epsilon_{\vec{k},\omega}$. They represent collective density waves, for $k < k_D$ which is the usual experimental situation, at the longitudinal phonon frequency ω_l, the plasma hybrid frequency $(\omega_p^2+\omega_c^2)$, and the cyclotron harmonics modes $n\omega_c$, $n = 2, 3, 4$, etc.

The scattered intensity at ω_l and $(\omega_p^2+\omega_c^2)^{1/2}$ is proportional to $(k/k_D)^2$ while the intensity at $n\omega_c$ is proportional to $(k/k_D)^2\lambda^n$ for $\lambda < 1$. The interaction of the cyclotron waves with plasmons has been considered previously.[10] We would like to consider here the interaction of longitudinal phonons with cyclotron waves. In order to estimate the strength of this interaction we consider the limiting case $\lambda \ll 1$. To dominant order in λ, Eq. (4) reduces (for $\tau \to \infty$):

$$\epsilon_{\vec{k}\omega} = \epsilon_\infty \frac{\omega^2-\omega_l^2}{\omega^2-\omega_t^2} - \omega_p^2\left(\frac{1-\lambda}{\omega^2-\omega_c^2}+\frac{\lambda}{\omega^2-4\omega_c^2}\right). \quad (5)$$

Here, the cyclotron mode-phonon interaction is most pronounced when the phonon frequency ω_l is about equal to $2\omega_c$, the eigenfrequency of the electron-cyclotron harmonic mode. The two eigenmodes will now split and share their intensity. The experimental observation of this interaction is possible if the frequency splitting is larger than $1/\tau$. In order to estimate the splitting, we define

$$X_0 = \frac{1-\lambda}{\omega^2-\omega_c^2}\,\omega_p^2 \approx \frac{1-\lambda}{\omega_l^2-\omega_c^2}\,\omega_p^2\,,$$

and using Eq. (5) solve for the approximate roots at ω_l and $2\omega_c$, where $\omega_l \approx 2\omega_c$, $\lambda \ll 1$, and $\omega_p \ll \omega_l$. In that limit the two roots come close to one another when

$$\omega_l^2 + (\omega_t^2 - 4\omega_c^2)X_0^2 = 4\omega_c^2 + \lambda\omega_p^2\,,$$

with a frequency split of

$$\Delta\omega = (\lambda)^{1/2}\omega_p\alpha/(1-X_0^2)\,, \quad (6)$$

where $\alpha^2 = (\omega_l^2-\omega_t^2)/\omega_l^2$. We next estimate $\Delta\omega$ for GaAs where $\alpha^2 = 0.2$ with $k/k_D = \frac{1}{2}$, $\omega_p = \omega_c$, and obtain $\Delta\omega/\omega_p \approx 0.25$. We therefore expect that for $\omega_p\tau \sim 10$ it will be possible to observe the splitting due to cyclotron mode-phonon interaction. Numerical computations of the roots and of $\text{Im}\,\epsilon_{\vec{k}\omega}^{-1}$ are given in Figs. 1 and 2, respectively. In Fig. 1 the dispersion relation for the coupled phonon-cyclotron harmonic modes for fixed $k/k_D = 0.5$ is plotted. The splitting at the crossover point is $\Delta\omega/\omega_p \approx \frac{1}{4}$, which agrees with the approximate estimate Eq. (6) Since the splitting is of interest from the experimental point of view, it is important to know how a finite collision time τ effect it. In Fig. 2 we have plotted $\text{Im}\,\epsilon_{\vec{k}\omega}^{-1}$, which is proportional to the Raman cross section, choosing $\omega_p\tau = 10$ for several values of ω_c. Here the phonon and the cyclotron harmonic lines can be clearly observed as well as the plasma-hybrid line. The intensity at the cyclotron mode is clearly decreased when its frequency moves away from the phonon line [Figs. 2(a) and 2(c)]. At the

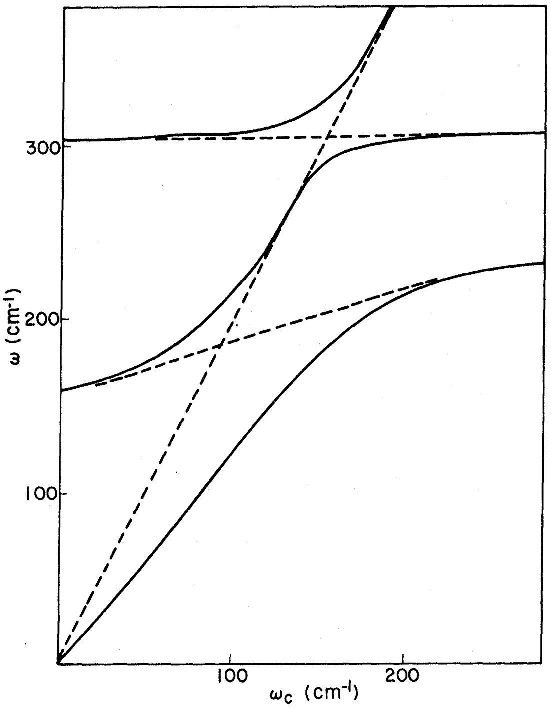

FIG. 1. Frequency of coupled phonon-magnetoplasmon–second-harmonic modes as a function of ω_c. Here $\omega_l = 295$ cm^{-1}, $\omega_t = 265$ cm^{-1}, $\omega_p = 150$ cm^{-1}. Solid lines are the dispersion curves with $k/k_D = 0.5$; dashed lines are the dispersion curves for $k/k_D = 0$.

crossover point [Fig. 2(b)] the phonon and cyclotron mode share their intensity, as expected. For a sample with $\omega_p\tau \gtrsim 5$ the splitting cannot be observed directly. One rather should observe the widening of the phonon line at the crossover point.

III. CALCULATION OF REFLECTIVITY

We calculate the response of the plasma-phonon system in a static magnetic field $\vec{B}_0$ to a radiation field, which is given by the solution of the Maxwell equations in our medium:

$$\vec{\nabla}\times\vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}, \quad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t} . \tag{7}$$

The electron current is related to the electric field by means of Ohms law

$$\vec{J} = \sigma\vec{E} . \tag{8}$$

Here $\vec{\sigma}$ is the electric conductivity which is connected to the dielectric tensor $\vec{\epsilon}$ via the relation

$$\vec{\epsilon} = \epsilon_\infty(\omega^2 - \omega_l^2)/(\omega^2 - \omega_t^2) + (4\pi i/\omega)\vec{\sigma} , \tag{9}$$

where $\epsilon_\infty(\omega^2 - \omega_l^2)/(\omega^2 - \omega_t^2)$ represents the polarizability of the lattice.

The dispersion relation of the electromagnetic field is obtained from Eqs. (7) and (8) to be

$$\left| k^2\delta_{\alpha\beta} - k_\alpha k_\beta - (\omega^2/c^2)\epsilon_{\alpha\beta}\right| = 0 . \tag{10}$$

We consider a Faraday configuration in which $\vec{k} \parallel \vec{B}_0$ for the right-hand circularly polarized electromagnetic field. For the local limit, which is a good approximation in one case, we obtain the dispersion relation to be

$$\frac{k^2c^2}{\omega^2} = \epsilon^R(\omega) = \epsilon_\infty\frac{\omega^2 - \omega_l^2}{\omega^2 - \omega_t^2} - \frac{\omega_p^2}{\omega(\omega - \omega_c + i/\tau)} , \tag{11}$$

where $\epsilon^R(\omega)$ is the right-hand polarization dielectric function and τ is the phenomenological electronic lifetime. The frequency ranges, in which no propagating solution exists, are obtained from the zeros and the poles of $\epsilon^R(\omega)$. The poles are given by $\omega = \omega_t$ and $\omega = \omega_c$, however, the zeros must be evaluated numerically for finite ω_c. It is easily found that for $\omega_c = 0$ the zeros are given by

$$\omega_{1,2}^2 = \tfrac{1}{2}\{\omega_l^2 + \omega_p^2 \pm [(\omega_l^2 - \omega_p^2)^2 + 4\omega_p^2(\omega_l^2 - \omega_t^2)]^{1/2}\}, \tag{12}$$

and for $\omega_c \to \infty$ they are at $\omega_1 = \omega_c$ and $\omega_2 = \omega_l$. The

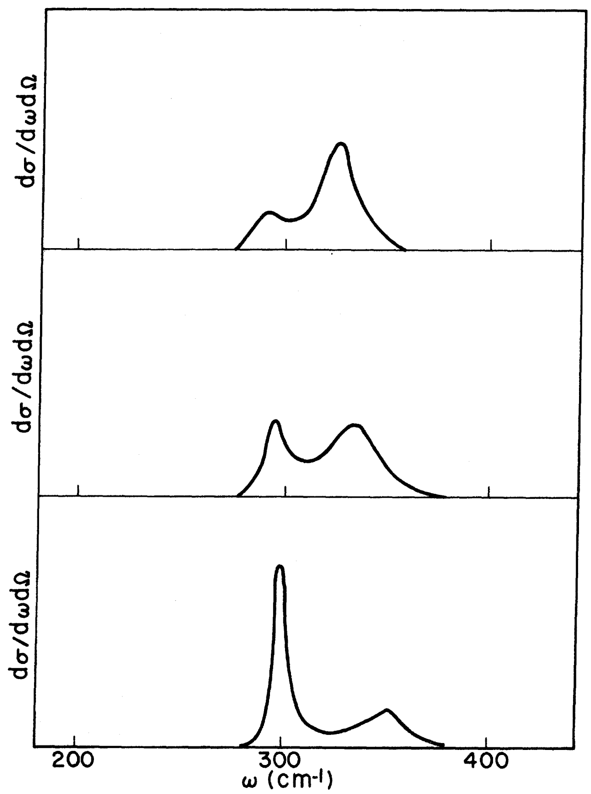

FIG. 2. Spectra of relative Raman-scattering cross section near the phonon frequency for the system specified in Fig. 1. Here $\omega_p\tau = 10$ and the cross section given in arbitrary units.

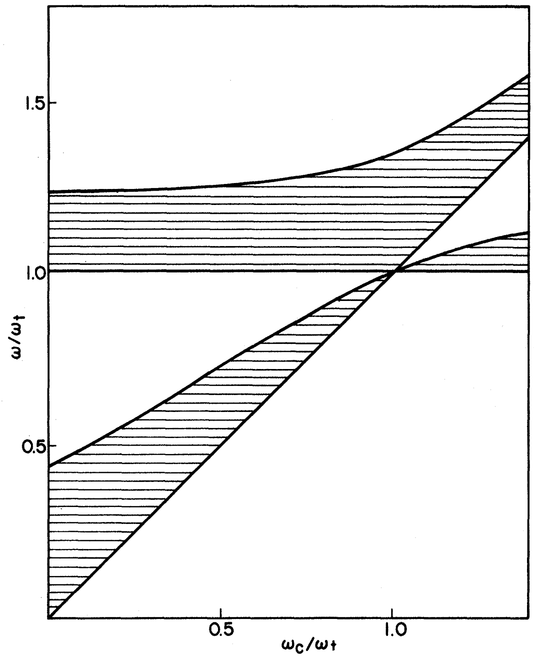

FIG. 3. Forbidden energy gaps as a function of ω_c. Shaded areas represent the gap. Here $\omega_l/\omega_t = 1.2$ and $\omega_p/\omega_t = 0.5$.

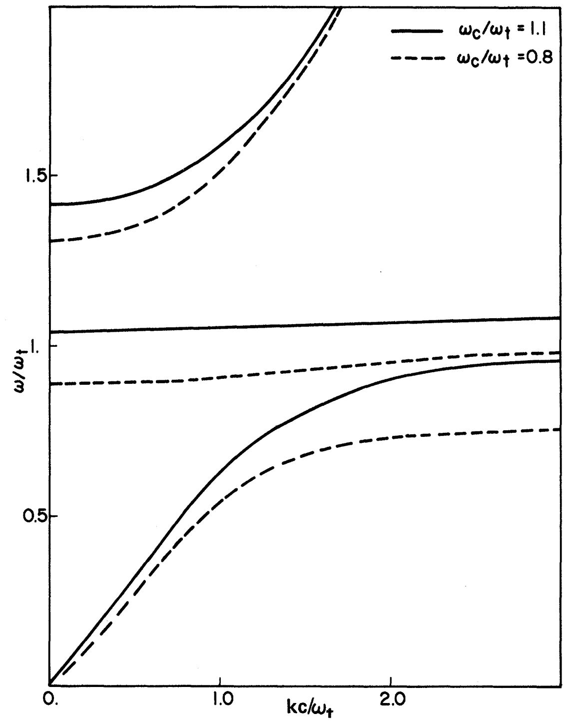

FIG. 4. Dispersion relation $\omega(k)$ for the coupled phonon-magnetoplasma system for $\omega_c/\omega_t = 1.1$ (solid line) and $\omega_c/\omega_t = 0.8$ (dashed line). Here $\omega_p/\omega_t = 0.5$ and $\omega_l/\omega_t = 1.2$.

numerical solution of the zeros of $\epsilon^R(\omega)$ as a function of the magnetic field has been evaluated and are shown in Fig. 3. Here the forbidden-frequency bands are shown by the shaded area. The interaction of the cyclotron wave with the transverse phonon is most pronounced when $\omega_c \sim \omega_t$, as expected. The dispersion relation of the coupled cyclotron-phonon modes is computed from Eq. (1) and is given in Fig. 4. Here for $\omega_c < \omega_t$ there are three eigenmodes; low-frequency cyclotron waves and coupled hybrid-plasma and phonon modes. However, when $\omega_c \sim \omega_t$ the cyclotron mode frequency approaches ω_t from below (for $k \to \infty$). For $\omega \gtrless \omega_t$ we obtain a narrow forbidden-frequency band above ω_t and a small allowed band due to (so to speak) isolated transverse phonon modes. Further increase of ω brings us to a large forbidden-frequency band. Experimentally the spectrum of coupled phonon-cyclotron modes can be obtained from reflectivity measurements. Using our plane-wave solution at the interface of the solid we obtain for the reflectivity R,

$$R = \left| \{1 - [\epsilon^R(\omega)]^{1/2}\} / \{1 + [\epsilon^R(\omega)]^{1/2}\} \right|^2 . \qquad (13)$$

The computed reflectivity as a function of ω_c is shown in Figs. 5 and 6. The top trace of Fig. 5

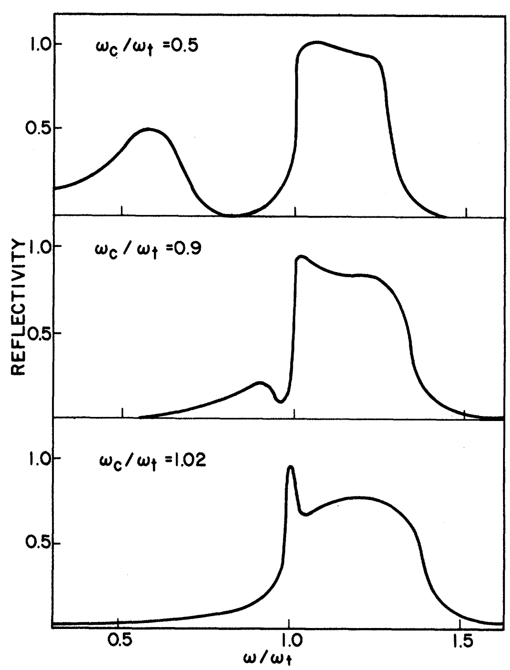

FIG. 5. Reflectivity of right-hand circular-polarized electromagnetic waves propagating along a strong dc magnetic field. Here $\omega_l/\omega_t = 1.2$, $\omega_p\tau = 10$, and $\omega_p/\omega_t = 0.5$.

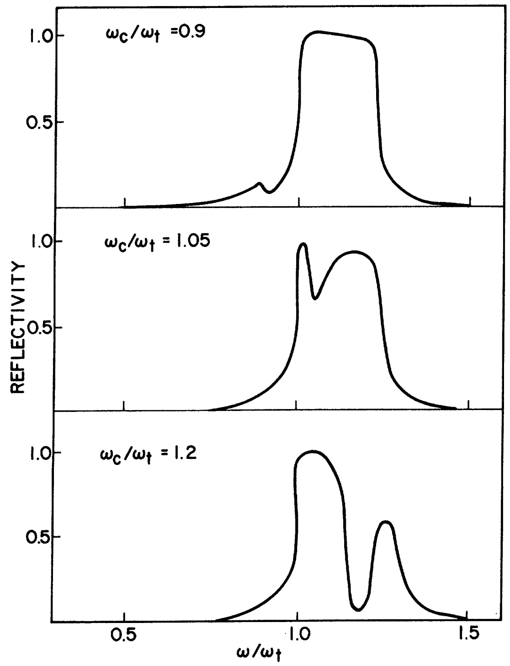

FIG. 6. Reflectivity of right-hand circular-polarized electromagnetic waves propagating along a strong dc magnetic field. Here $\omega_l/\omega_t = 1.2$, $\omega_p\tau = 10$, and $\omega_p/\omega_t = 0.2$.

shows the existence of the two forbidden gaps which are separate from one another (here $\omega_c/\omega_t = 0.5$). When $\omega_c \approx \omega_t$, a narrow forbidden gap appears in the vicinity of ω_t as clearly indicated by the lower traces of Fig. 5. In Fig. 6 we repeat the calculations for the reflectivity for smaller plasma frequency ω_p. Here again we see changes in the reflectivity as ω_c sweeps through ω_t. The upper trace, for $\omega_c/\omega_t = 0.9$, shows a stop band above ω_t, due to the lattice polarizability, as well as a regime of increased reflectivity below ω_t. The middle trace, for $\omega_c/\omega_t = 1.05$, shows the very narrow stop band immediately above ω_t. The lower trace, for $\omega_c/\omega_t = 1.2$, shows the allowed frequency band above ω_t induced by the coupling of the phonons to the cyclotron waves.

In conclusion, the reflectivity measurements will clearly exhibit the strong interaction between cyclotron and phonon modes which occurs in the vicinity of $\omega_c \approx \omega_t$. This consists of a drastic change in the allowed regimes of propagation: For $\omega_c \gtrless \omega_t$, the modes give rise to two distinct stop bands, whereas near $\omega_c \approx \omega_t$ they acquire a fine structure consisting of narrower regions of alternating stop and pass bands.

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